

**MTH 203, Calculus III, Quiz one**

Ayman Badawi

**QUESTION 1.** a) Find a vector in 2D, say  $v$ , where  $|v| = 3.4$  and it is perpendicular to the vector  $w = -3i + 4j$ .

**QUESTION 2.** Given  $q_1 = (1, 1, 1)$ ,  $q_2 = (-1, 1, 1)$ ,  $q_3 = (0, 1, 2)$  are three points in 3D.

a) Is there a unique plane containing  $q_1, q_2, q_3$ ? Explain

b) If the answer is yes for part (a), then find the equation of the plane.

**QUESTION 3.** Given  $(1, 1, 1)$  lies on the line  $L$  that is perpendicular to the plane  $-2x + y + z = -7$ . Find the parametric equations of  $L$ . Since  $L$  intersects the given plane in one point say  $Q$ , find  $Q$ .

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**MTH 203, Calculus III, Quiz two**

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**QUESTION 1. (Circle the correct answer)**

- (i) Given  $V_1$  and  $V_2$  are vectors having the same initial point where  $V_1 = 2i - 2j + k$ ,  $V_2 = i + j - 3k$ . Then  $|proj_{V_1} V_2| =$   
 a)  $\frac{-3}{9}$  b)  $\frac{3}{9}$  c) 1 d) None
- (ii) Let  $V_1, V_2$  as in the previous question. Then  $proj_{V_1} V_2 =$   
 a)  $-2i + 2j - k$  b)  $\frac{2}{3}i - \frac{2}{3}j + \frac{1}{3}k$  c)  $-\frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k$  d) None
- (iii) One of the following vectors lies in the plane  $2x + y + z = 12$   
 a)  $\langle 1, 0, 10 \rangle$  b)  $2i + j + k$  c)  $-i + 2k$  d)  $\langle -2, 0, -4 \rangle$
- (iv) One of the following is a standard form of the plane  $3x - 2y + 5z = 30$  is  
 a)  $3x - 2y + 5z - 20 = 0$  b)  $3x - 2(y - 10) + 5(z - 10) = 0$  c)  $3(x - 10) - 2y + 5(z - 6) = 0$  d) None
- (v) One of the following planes is perpendicular to the plane  $3x - y + z = 1$   
 a)  $-2x + 6z = 30$  b)  $6x - 2y + 2z = 0$  c)  $-x + 3y = 1$  d) none
- (vi) One of the following lines is parallel to the plane  $x + y + z = 10$   
 a)  $x = 2t, y = t, z = 1 - 3t$  b)  $bx = 2t, y = 4 + 6t, z = 6 - 8t$  c)  $x = 2t, y = t, z = 4 - 2t$  d) none
- (vii) Given  $2x - 1 = y + 4 = -z + 5$  are the symmetric equations of a line in 3D. Parametric equations of the line can be :  
 a)  $y = t, x = t + 5, z = 1 - t$  b)  $x = t, y = 2t - 5, z = 6 - 2t$  c)  $x = 2t - 1, y = t + 4, z = -t + 5$  d) None
- (viii) Given the parametric equations of two lines:  $L_1 : x = 1 + t, y = 3 + 2t, Z = 4 - t$  and  $L_2 : x = 1 - s, y = 5 + 2s, z = 4 - s$ . Then  
 a)  $L_1$  is parallel to  $L_2$  b)  $L_1$  is the same line as  $L_2$  c)  $L_1$  and  $L_2$  are screw lines d)  $L_1$  and  $L_2$  are interesting lines.
- (ix) Let  $V_1, V_2$  be vectors in 3D having the same initial point. Given  $V_1 \times V_2 = \sqrt{2}i + j + k$ , the angle between  $V_1$  and  $V_2 > 90$ , and  $|V_1| = |V_2| = 2$ . Then  $V_1 \cdot V_2 =$   
 a) -2 b) -4 c)  $-\sqrt{3}$  d)  $-2\sqrt{3}$  e) Cannot be determined.
- (x) Given the two planes  $x + y + z = 4$  and  $-x - y = 2$  intersect in a line  $L$ . Parametric equation of  $L$  can be :  
 a)  $x = -t, y = 2 + t, z = 2$  b)  $x = t, y = 2 - t, z = 2$  c)  $x = 2t, y = -2t, z = 4$  d)  $x = t, y = -2 - t, z = 6$   
 e) None

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**MTH 203, Calculus III, Quiz Three**

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**QUESTION 1. (Circle the correct answer)**

- (i) Given  $Q = (\sqrt{8}, 1)$  is not on the line  $L : y = \sqrt{8}x + 5$ . The distance between  $Q$  and the line  $L$  is :  
a) 3   b) 2   c)  $2/3$    d) 4
- (ii) Given  $Q = (0, 2, -2)$  is not on the line  $L : x = t, y = 2 + 2t, z = 2 - 2t$ . The distance between  $Q$  and  $L$  is  
a)  $\sqrt{5}/9$    b)  $\sqrt{5}/3$    c)  $2\sqrt{5}$    d)  $4\sqrt{5}/3$
- (iii) An equation of the plane where each point in the plane is equidistant from the two points  $Q_1 = (1, 3, 0)$  and  $Q_2 = (3, 1, 2)$   
a)  $-2x + 2y - 2z = -2$    b)  $2x - 2y + 2z = -4$    c)  $x - y + z = 3$    d)  $x - y + z = 4$
- (iv) Given  $P_1 : \sqrt{6}x - 3y + 7z = 0$  and  $P_2 : -2\sqrt{6} + 6y - 14z = -64$  are non-intersecting planes. Distance between  $P_1$  and  $P_2$  is  
a) 4   b) 8   c) 1   d) 2
- (v) Let  $V$  be a vector where  $V = \langle \sqrt{11}, -3, 4 \rangle$ . Then  $V$  makes an angle with the positive  $x$ -axis equals to  
a) 30   b) 120   c) 150   d) 60
- (vi) Given  $Q_1 = (1, -5), Q_2 = (4, -6), Q_3 = (2, -8)$  are the vertices of a triangle in  $2D$ . The area of the triangle is  
a) 5   b) 4   c) 8   d) 5
- (vii) Given two lines  $L_1 : x = 0.5t, y = 2 + t, z = -2 - t$  and  $L_2 : x = s, y = 2 + 2s, z = 2 - 2s$  where  $L_1$  and  $L_2$  are non-intersecting PARALLEL lines. The distance between  $L_1$  and  $L_2$  is  
a)  $2\sqrt{5}$    b)  $\sqrt{5}/3$    c)  $4\sqrt{5}/3$    d) Cannot be determined
- (viii) Given  $v_1 = 2i + j - 6k, v_2 = 4i + j, v_3 = -i + 3j$  (all having the same initial point in 3D) form a parallel-piped object in 3D. The volume of the object is  
a) 13   b) 66   c) 78   d) 11

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**MTH 203, Calculus III, Quiz FIVE**

Ayman Badawi

**QUESTION 1. (Circle the correct answer)**

- (i) Let  $r(t) = \langle 2, \sin(2t), \cos(2t) \rangle$ . The Unit normal vector at  $t = \pi/8$  is  
a)  $\langle 0, \sqrt{2}/2, -\sqrt{2}/2 \rangle$     b)  $\langle 0, -\sqrt{2}/2, -\sqrt{2}/2 \rangle$     c)  $\langle 0, -\sqrt{2}/2, \sqrt{2}/2 \rangle$
- (ii) Let  $r(t) = \langle 4t, 3t, 5 \rangle$ . Then the arc-length of  $r(t)$  when  $t$  is between 1 and 6  
a) 25    b) 30    c) 125    d) 150
- (iii) Let  $r(t) = \langle \sqrt{16-t^2}, \sqrt{t-1}, \frac{1}{\sqrt{3-t}} \rangle$ . The domain of  $r(t)$  is  
a)  $(-4, 4) \setminus \{3\}$     b)  $[1, 3)$     c)  $(-\infty, 4) \setminus \{3\}$     d)  $[1, \infty) \setminus \{3\}$
- (iv) Given  $r'(t) = \langle 6t(1+t^2)^2, \frac{-e^t}{2-e^t}, 2e^{2t} \rangle$  and  $r(0) = \langle 4, 5, -2 \rangle$ . Then  $r(t)$   
a)  $\langle (1+t^2)^3 + 3, \ln|2-e^t| + 5, e^{2t} - 3 \rangle$     b)  $\langle (1+t^2)^3, \ln|2-e^3| + 4, e^{2t} - 2 \rangle$   
c)  $\langle 2(1+t^2)^3 + 2, -\ln|2-e^t| + 5, 2e^{2t} - 4 \rangle$
- (v) The arc-length of  $r(t) = \langle 2\sin(t), \sqrt{2}\cos(t), \sqrt{2}\cos(t) + 1 \rangle$  when  $t$  is between 0 and  $\pi/2$  is  
a)  $2\pi$     b)  $\pi$     c)  $\pi/2$     d)  $\sqrt{2}\pi/2$

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## MTH 203, Calculus III, Quiz Six

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**QUESTION 1.** (i) Let  $f(x, y) = 2x^2 + e^y + yx$ . Find  $D_u(1, 0)$  in the direction of  $v = 3i + 4j$ . What is the maximal value of  $D_u(1, 0)$ ? and in which direction does the max of  $D_u(1, 0)$  occur?

(ii) Find equations for the tangent plane and the normal line to the surface  $e^z + x^2y + y = 3$  at  $(1, 1, 0)$

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**MTH 203, Calculus III, Quiz SEVEN**

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**QUESTION 1.** (i) Let  $D = \{(x, y) \mid x^2 + y^2 \leq 9 \text{ where } x, y \geq 0\}$  (i.e.,  $D$  is a quarter of a circle of radius 3 in the first quadrant of the  $xy$ -plane). Find the volume of the solid object that is determined by  $f(x, y) = x^2y$  over  $D$

(ii) Determine the surface area of the solid object that is determined by  $f(x, y) = x^3 + 2y$  over the region  $D$  in the  $xy$ -plane that is bounded by the  $x$ -axis and  $y = x^3$ .

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